Chapter 18

Spatial analysis based on cost functions

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Abstract

Each model of past movement based on the historical and archaeological evidence nowadays relies implicitly or explicitly on a cost function estimating costs of movement in terms of time, calories or some other currency for the study area and period of time considered. Cost functions are the basis of two popular GIS-based areas of spatial analysis in archaeology: site catchments and least-cost paths. A site catchment is the region accessible from a site, and often archaeological studies analyse the resources within this region. A least-cost path (LCP) is the route minimizing the costs of movement between two given locations. In archaeological studies, the aim of LCP calculations is often to reconstruct ancient routes or a route network or to identify the principal factors governing the construction of known roads or road segments. For generating a site catchment, GIS programs compute an accumulated cost surface (ACS). This is also the first step for LCP generation, therefore the Dijkstra algorithm implemented in most GIS programs for this purpose is outlined. The cost function applied for calculating the ACS typically depends on slope, soil type, vegetation cover or the presence of streams. Several slope-dependent cost functions and a list of terrain factors are presented. Moreover, issues with least-cost applications are discussed, including barrier implementation, restricted number of possible movement directions, and the calculation of slope-based costs for a path taken in one or two directions. The aim of the case study is to compare the site catchments of a Roman farm and a temple at different scales based on a cost model derived from Roman roads that were recorded in three publications. The concluding section discusses validation and stability of the outcomes of the methods presented and some additional issues such as modelling movement beyond walking. Finally, some other cost function based approaches are outlined.

Introduction

Many archaeologists are no longer satisfied with presenting distribution maps, but their aim is the identification of the patterns of movement that explain how the people and the artefacts of the time period considered got to the sites (e.g. Rademaker, Reid, & Bromley, 2012). For most regions and periods of time, the distribution of artefacts provides the most important evidence for the movement of people. Archaeological remains of ancient paths, roads and ship wrecks are fairly rare, and in most cases indicate only small sections of the original trajectory. Moreover, dating land routes is often difficult due to continuous use after initial path creation and absence of diagnostic finds. However, the archaeological record of human movement can sometimes be supplemented by historical sources.

Each model of past movement based on the historical and archaeological evidence nowadays relies implicitly or explicitly on a cost function estimating costs of movement in terms of time, calories or some other currency for the study area and period of time considered. Evidence for the popularity of such approaches in archaeology are not only numerous case studies published since 2000 but also several sessions at the annual Computer Applications in Archaeology (CAA) conference dealing with this subject as well as two edited volumes with contributions focusing solely on least-cost methods or applications (Polla & Verhagen, 2014; White & Surface-Evans, 2012).

Two popular GIS-based areas of spatial analysis in archaeology are based on cost functions: site catchments and least-cost path. A site catchment is the region accessible from a site, and often archaeological studies analyse the resources within this region (Conolly & Lake, 2006, p. 214). A least-cost path (LCP) ideally is the route minimizing the costs of movement between two given locations (Conolly & Lake, 2006, pp. 294, 252–255).

In fact, the most basic application of a cost function is the generation of a least-cost site catchment (LCSC). The LCSC includes all areas that can be reached by expending less than a user-selected cost limit. The term isochrone is often used for the boundary when costs are measured in terms of time. According to Wheatley and Gillings (2002, p. 159), the concept of LCSC was derived from defining the exploitation territory of a site. Beyond the boundary of this territory the costs of exploitation exceed the benefit. This concept is closely related to time geography introduced by Mlekuz (2013) into archaeological least-cost modelling. Site catchment analysis for foraging societies mainly focuses on the types and quantities of resource areas within each catchment zone (e.g. Surface-Evans, 2012). Most publications presenting site catchments for sedentary agrarian cultures study the potentials for crop production, in terms of soil, topography, slope etc. (e.g. Korczyńska, Cappenberg, & Kienlin, 2015). Wheatley and Gillings (2002, p. 160) refer to a widely cited paper published in 1970 suggesting a cost limit of 1 hour walk for a sedentary agricultural site and 2 hours for a herding/hunting community. LCSC derived from several cost limits may be appropriate, if each settlement is surrounded by rings of different utilisation, as was described in the early 19th century by von Thünen's model of rural land use (Waugh, 2002, pp. 471–475). For instance, Posluschny (2010) uses the popular Tobler hiking function (Tobler, 1993) with two time limits, 60 and 15 minutes, with 15 minutes delimiting the area of daily farming activities. For Posluschny's study area in south western Germany, early Iron age settlements with overlapping catchments on the 15 minute scale are probably not contemporary. So catchment overlap may indicate some issues with dating or the cost limit selected. The aim of Gaffney and Stančič (1992) was to define realistic mutually exclusive exploitation areas for the seven principal hillforts on the island of Hvar, Croatia. Therefore, they calculated LCSC based on a 90 minute walking time limit. For a project reconstructing land use patterns of sites, catchments may define the survey area (Peeples, Barton, & Schmich, 2006). Comparing catchment sizes may provide insights into the function of settlements. For instance, the study of Posluschny (2010) mentioned above compares the catchment sizes for early Iron Age princely sites with those of non-princely settlements of the same period and comes to the conclusion that agriculture was more important for the normal settlements. Site catchment analysis was introduced by processual archaeology (Conolly & Lake, 2006, p. 209) focusing on economic costs although it is also possible to include social aspects such as visibility or taboo zones in a cost model (Table 18.1). Lee and Stucky (1998) provide a comprehensive overview of approaches for including viewsheds in least-cost calculations. As the LCSC comprises all LCPs that expend the catchment's cost limit or less, it can be spoken of as the 'potential path area' (Mlekuz, 2013).

In archaeological studies, LCPs often provide reconstructions of ancient routes or route sections (e.g. Chapman, 2006, pp. 110 –111; Herzog, 2013e; Rademacher et al., 2012; Verhagen & Jeneson, 2012), take for example Rogers, Collet, and Lugon (2015) who calculate LCPs in an attempt to predict high mountain passes in prehistoric times. LCPs may also be applied to identify the principal factors governing the construction of known roads or road segments (e.g. Bell & Lock, 2000; Fovet & Zakšek, 2014; Güimil-Fariña & Parcero-Oubiña, 2015; van Lanen, 2017, pp. 123–134). If LCPs coincide with known roads only after forcing the LCP to visit an intermediate location, this is evidence for the importance of this additional node (e.g. Güimil-Fariña & Parcero-Oubiña, 2015).

In most studies, a set of points is connected by LCPs (e.g. Canosa-Betés, 2016). Alternatively, LCPs in all directions can be constructed starting from a given site, resulting in focal mobility networks (Fábrega Álvarez & Parcero Oubiña, 2007; Herzog, 2013c; Llobera, Fábrega-Álvarez, & Parcero-Oubiña, 2011; Lynch & Parcero Oubiña, 2017). If the movement costs depend on topography, topographic data (mostly a digital elevation or surface model) with adequate resolution is required, whereas in ancient cities, each house is a barrier and should be modelled accordingly (Branting, 2007). Often the reconstruction of past routes by LCPs is the basis of further research, e.g. Hudson (2012).

Reference	Slope cost component	Additional cost components
Canosa-Betés (2016)	Tobler (1993); walker cost function of Llobera and Sluckin (2007); Herzog (2013a based on Minetti, Moia, Roi, Susta, and Ferretti (2002)	4 categories of water courses (breadth: 200 m, 150 m, 50 m, and 25 m); no extra costs for possible locations of fords or bridges
Fovet and Zakšek (2014)	3rd degree polynomial based on Minetti et al. (2002)	Visibility, based on a variable similar to sky view
Güimil-Fariña and Parcero-Oubiña (2015)	Tobler (1993); Pandolf, Givoni, and Goldman (1977); Herzog (2013a based on Minetti et al., 2002); walker cost function of Llobera and Sluckin (2007)	Penalty for crossing rivers equivalent to ascending a 15° gradient
Groenhuijzen and Verhagen (2017)	Velocity estimate derived from Pandolf et al. (1977) assuming constant values for metabolic rate, weight and load.	Terrain coefficients based on Soule and Goldman (1972); coefficient 20 for rivers and streams
Herzog (2013e)	Vehicle cost function. Herzog (2013a)	Avoiding wet soils including streams; lower costs for fords
Korczyńska et al. (2015)	Tobler (1993)	none
Van Lanen (2017)	Slope classes based on natural breaks; slopes > 10% are considered impassable	Terrain classification: factor 1.2 for higher sandy heath land, 1.8 for lower wetlands; groundwater level
Lynch and Parcero Oubiña (2017)	Walker cost function of Llobera and Sluckin (2007)	Impedance factor 2 for areas from which no high mountain top is visible.
Posluschny (2010)	Tobler (1993)	none
Rademaker et al. (2012)	Pandolf et al. (1977) with various values for variables W, L, V (see Table 18.2)	Terrain coefficients based on Soule and Goldman (1972)
Rogers et al. (2015)	Tobler (1993); alternatively: Swiss 15th degree polynomial	landcover
Surface-Evans (2012)	Tobler (1993)	none
Verhagen and Jeneson (2012)	Tobler (1993)	Alternative to slope: Visibility based on low-pass filtered openness

Table 18.1: Cost components applied in selected archaeological least-cost studies published in 2010 or later.

The sites of many cultural groups prefer locations close to ancient roads or paths (e.g. Fovet & Zakšek, 2014), therefore, successful road reconstruction often allows predictive modelling of road-related sites such as *mansiones*, i.e. resting places along Roman roads, but also of archaeological features such as rock art or burial mounds. Another focus has been on what might be termed 'natural' pathways in a given landscape. An early example for a site prediction approach based on pathway reconstruction using a cost model was presented by Bellavia (2002) who sought to derive "natural pathways" from a digital elevation model (DEM) in several areas of the UK including Stonehenge. Some evidence has also been published for animals travelling on least-effort routes (Ganskopp, Cruz, & Johnson, 2000), so if early hunters followed the paths of animals as has been suggested by some authors (e.g. Whitley & Burns, 2008) they most probably walked on LCPs.

An appropriate cost function determines the costs of movement in the region studied and should take the means of transportation available to the people living at that time into account, e.g. the use of pack or draft animals, wheeled vehicles or boats. Nearly all archaeological case studies applying cost functions include the cost factor slope, often combined with factors depending on soil, land use, the presence of streams, or visibility (Table 18.1; cf. Herzog, 2014b for some additional archaeological LCSC and LCP publications).

Method

Overview

The initial step in LCSC and LCP calculation is the decision concerning the principal factors governing movement costs, and establishing an appropriate cost function combining the costs of these factors. The next step is the creation of an accumulated cost surface (ACS), that is a raster grid storing the costs of movement from the origin to every other cell in the raster grid. The ACS is normally calculated by spreading out from the origin and accumulating the costs of the cells as each is visited. For LCSC, the origin is the site location. For LCPs, the origin is one of the two locations to be connected. The LCSC is derived from the ACS by stopping the spreading process for cells whose accumulated costs exceed the predefined cost limit. These cells form the boundary of the catchment. Alternatively, an isoline at the cost limit value may be derived from the ACS. The LCP is derived from the ACS by backtracking from the target location to the origin. These three steps will be described in more detail in the next sections.

Finally, some validation and analysis of the stability of the outcomes should be included in each study creating LCSC or LCPs, this is discussed in the “Conclusion” section below.

Estimating movement costs

In archaeological case studies movement costs are typically measured in time or energy expenditure. The two measurement systems differ, and some authors give reasons for preferring energy expenditure to time costs (e.g. Rademaker et al., 2012) or vice versa. The best option may depend on the culture considered. Some studies are based on cost estimations using different units, such as percent or angle slope (e.g. Bell & Lock, 2000; Bellavia, 2002). In all applications of a cost function, validation of the function chosen should be part of the study.

Table 18.1 illustrates that the most popular cost component in recent archaeological least-cost studies is slope, with most of the cost functions applied and listed in Table 18.2 being rules of thumb rather than based on a large sample of measurements. Formulas derived from measurements of modern humans who do not walk as frequently as people of the period considered may not necessarily outperform cost functions based on practical experience. Some of the slope-dependent cost functions in Table 18.2 (including the most popular ones) were already discussed in Herzog (2013a).

Tobler (1993) proposed the most popular cost function (no. 1 in Table 18.2), but some care has to be taken to implement this formula properly because the original formula estimates velocity (and not time) and relies on mathematical slope which differs substantially from slope in percent or degrees (Herzog, 2014a). The modified Tobler function (no. 2 in Table 18.2) was suggested by Márquez-Pérez et al. (2017) based on GPS data for 21 trails located in Spain. The original Tobler estimates are about 1.35 faster than the modified version, with a low standard deviation of 0.064 (tested for steps of 1 percent in the range of -70 to +70 percent). The estimates provided by this function outperformed those of MIDE, Langmuir, and standard Tobler for 10 of the 21 trails. W. Tobler (personal communication, October 17, 2017) recommended the study by Irmischer and Clarke (2017), the formulas presented in this study (no. 3 in Table 18.2) rely on data collected from 200 volunteer cadets between 17 and 23 years of age. Compared to Tobler’s formula that refers to soldiers hiking a known route, a lower average speed is recorded in this study, which is attributed by the authors to way finding costs.

No.	Name / Reference	Formula	Properties
1	Tobler (1993)	V(s) = 6*exp(-3.5*abs(s+0.05)) cost(s, ΔD) = 60*(ΔD / V(s))	V(s) estimates the velocity (km/h) on a gradient, cost(s, ΔD) estimates the time in minutes for covering the distance ΔD in km on a gradient with slope s.
2	Márquez-Pérez, Vallejo-Villalta, and Álvarez-Francoso (2017)	V(s) = 4.8*exp(-5.3*abs((s*0.7)+0.03)) V(s) = 4.8*exp(-3.71*abs(s+0.04286))	Modified Tobler: first formula as published, second formula is equivalent except for round-off errors
3	Irmischer and Clarke (2017)	V_{on}(ŝ) = f*(0.11 + exp(-(ŝ +5)²/1800))) V_{off}(ŝ) = f*(0.11 +0.67*exp(-(ŝ +2)²/1800)))	V_{on}(ŝ) estimates the on-road velocity (m/s) of walkers, V_{off}(ŝ) refers to off-road movement, f = 1.00 for male and f = 0.95 for female walkers.
4	Garmy, Kaddouri, Rozenblat, and Schneider (2005)	V(α) = 4*exp(-0.008*α²)	α is slope in degrees.
5	Langmuir (2004); implemented in r.walk (GRASS)	cost(Δd, ΔH_up, ΔH_gd, ΔH_sd) = a*Δd + b*ΔH_up + c*ΔH_gd + d*ΔH_sd Langmuir: a=0.72, b=6.0, c=1.9998, d=-1.9998 Downhill default slope value threshold is at 21.25%	Estimates time in seconds. All Δ values are in m. Δd = horizontal distance covered ΔH_up = positive height change ΔH_gd = gentle descent ΔH_sd = steep descent
6	Ericson and Goldstein (1980)	cost(Δd, ΔH_up, ΔH_dn) = Δd + 3.168*ΔH_up + 1.2*abs(ΔH_dn)	Δd and ΔH_up as in row 5 ΔH_dn = negative height change
7	MIDE: París Roche (2008, p. 11)	cost(N, Δd, ΔH_up, ΔH_dn) = N*0.012*Δd + 0.15*ΔH_up + 0.1*abs(ΔH_dn)	Δd, ΔH_up, ΔH_dn as in row 6 Estimates time in minutes. N is a terrain factor
8	Bellavia (2002)	Cost(N, α) = N * (abs(α)+1)	α is slope in degrees. N is a terrain factor.
9	Vehicle cost function. Herzog (2013a) based on Llobera and Sluckin (2007)	Cost(ŝ) = 1 + (ŝ / š)² The abbreviation Q(š) refers to this cost function, Q is short for quadratic.	š is the critical slope, i.e. for slopes exceeding š, hairpin turns are more effective than direct ascent or descent.
10	Llobera and Sluckin (2007)	Cost(s) = 2.635 + 17.37*s + 42.37*s² - 21.43*s³ + 14.93*s⁴	Walker cost function: Estimates energy consumption in kJ/m
11	Herzog (2013a) based on Minetti et al. (2002)	Cost(s) = 1337.8*s⁶ + 278.19*s⁵ - 517.39*s⁴ - 78.199*s³ + 93.419*s² + 19.825*s + 1.64	Walker cost function: Estimates energy consumption in kJ/(m*kg)
12	Pandolf et al. (1977)	Cost(W,L,N,V, ŝ) = 1.5*W + 2.0*(W+L) * (L/W)² + N*(W+L)*(1.5*V² + 0.35*V*|ŝ|)	Estimates metabolic rate in watts. W = weight (kg), L = load (kg), N = terrain factor, V = velocity (m/s)

Table 18.2: Slope-dependent cost functions, with ŝ percent slope, and s = ŝ/100 mathematical slope. If Δd (or ΔD) is missing in the cost formula, the result of the cost formula is to be multiplied by the distance covered. Rows 1 to 7 list cost functions estimating time, the formulas in rows 10 to 12 estimate energy consumption. The cost functions listed in rows 8 and 9 measure abstract cost units, which can best be understood by comparing estimates resulting from movement on a gradient with that on level ground.

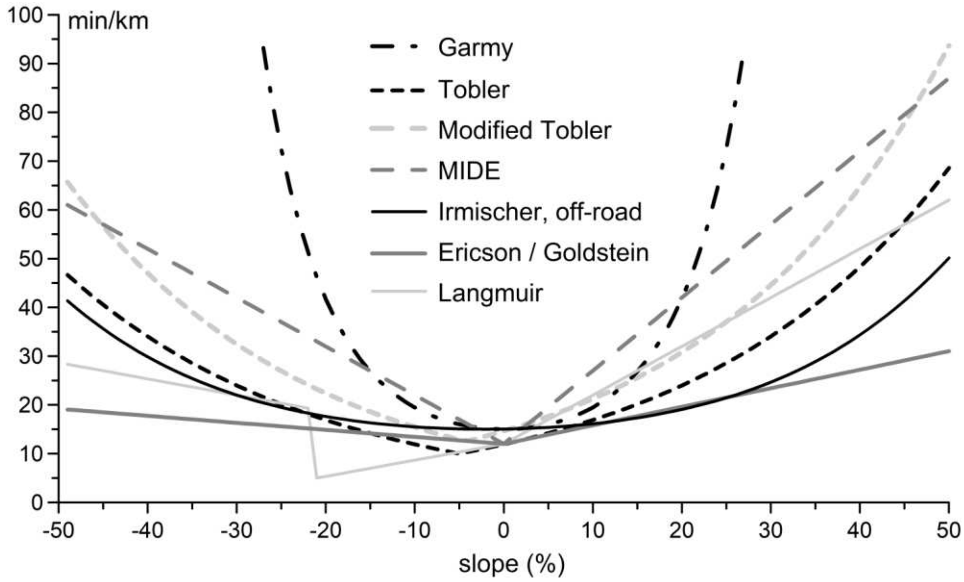


Figure 18.1: Cost functions estimating walking time; on the x-axis downhill slopes are negative.

Most of the slope-dependent cost functions are anisotropic, i.e. the costs for descending a gentle gradient are less than that of climbing a gentle slope (Figure 18.1). This may result in different optimal paths connecting two locations A and B, depending on the direction of movement. But most paths are used in both directions so that a cost function averaging the costs of movement in both directions seems appropriate in many situations. By averaging, the asymmetric cost curve is converted to a symmetric curve (Herzog, 2013a; Figure 18.2). Likewise, if the load carried by a descending walker, pack animal or vehicle differs from the load on the way up, the cost functions used should vary accordingly. Note that the slope-dependent cost functions do not include the costs for climbing stairs or ladders.

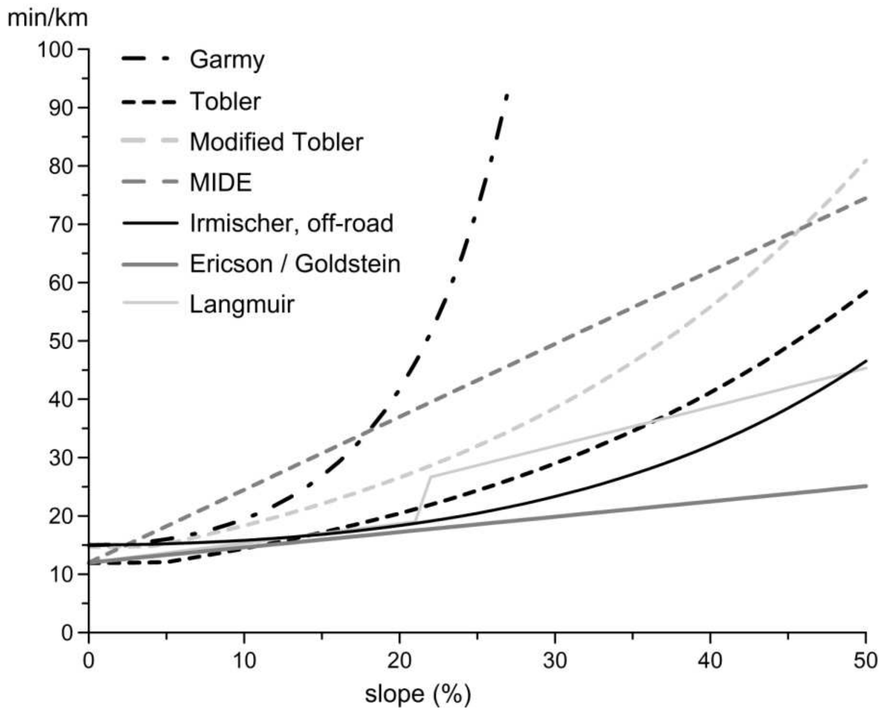


Figure 18.2: Cost functions estimating walking time: uphill and downhill costs are averaged.

Factor	Terrain	unit	Formula / reference
1.00	Blacktop roads and improved dirt paths	hour	MIDE: París Roche (2008), p. 11
1.00	Pavement (cement)	hour	De Gruchy et al. (2017)
1.03	Lawn grass	hour	De Gruchy et al. (2017)
1.19	Loose beach sand	hour	De Gruchy et al. (2017)
1.24	Disturbed ground (former stone quarry)	hour	De Gruchy et al. (2017)
1.25	horse riding paths, flat trails and meadows	hour	MIDE: París Roche (2008), p. 11
1.35	Tall grassland (with thistle and nettles)	hour	De Gruchy et al. (2017)
1.50	Open space above the treeline i.e. 2000 m above sea level	hour	Rogers et al. (2015)
1.67	bad trails, stony outcrops and river beds	hour	MIDE: París Roche (2008), p. 11
1.67	off-path	hour	Tobler (1993)
1.79	Bog	hour	De Gruchy et al. (2017)
2.00	Off-path areas below the treeline including pastures, forests, heathland, beaches etc.	hour	Rogers et al. (2015)
2.50	rock	hour	Rogers et al. (2015)
5.00	Swamp, water course	hour	Rogers et al. (2015)
1.00	Asphalt/blacktop	joule	De Gruchy et al. (2017)
1.10	Dirt road or grass	joule	De Gruchy et al. (2017)
1.20	Hard-surface road	joule	Givoni and Goldman (1971)
1.20	Light brush	joule	De Gruchy et al. (2017)
1.30	Ploughed field	joule	De Gruchy et al. (2017)
1.50	Ploughed field	joule	Givoni and Goldman (1971)
1.50	Heavy brush	joule	De Gruchy et al. (2017)
1.60	Hard-packed snow	joule	De Gruchy et al. (2017)
1.80	Swampy bog	joule	De Gruchy et al. (2017)
1.80	Sand dunes	joule	Givoni and Goldman (1971)
2.10	Loose sand	joule	De Gruchy et al. (2017)

Table 18.3: Published terrain factors for cost functions measuring time (unit: hour) or energy consumption (unit: joule) of a walker.

Several publications provide terrain factors that model reduced speed or energy consumption of a walker (Table 18.3). In many least-cost studies, water is considered as barrier, for instance Rogers et al. (2015) select factor 5 for traversing water courses and 499.5 for water bodies. In general, the effort needed for crossing a stream depends on many factors including width, depth and current (Langmuir, 2004, pp. 185–199).

The formula of Pandolf et al. (1977) shows one way of combining cost components, i.e. slope and a terrain factor. It consists of a term depending on weight and load only (estimating the energy consumption of standing) plus the extra energy required for movement, and only the latter term is multiplied by the terrain coefficient. The formula presented by Givoni and Goldman (1971), that is also used by Soule and Goldman (1972), is the product of the terrain coefficient with a factor depending on weight, load, velocity, and slope. Alternatively, a (weighted) sum of the two or more cost components may be applied (e.g. Fovet & Zakšek, 2014), if the cost components are independent. Additional approaches for combining cost components and their drawbacks are discussed by Herzog (2013a).

The time and energy consumption required for walking a path depend also on factors varying throughout the year and on weather conditions: Rain, muddy paths due to recent rain, snow, storm, fog, high humidity, very high or very low temperatures may slow down progress considerably. Moreover, movement costs depend on the sex, age, weight, load, and fitness of the walker as well as on the number of hikers in the walking group. Therefore, the stability of any least-cost result should be analysed by varying the model parameters.

Estimating the costs of movement by boat or ship is even more difficult than estimating the costs of walking, due to differences in boat or ship technology, seasonal variations, currents, and substantial changes of the rivers or coastlines since the period considered. Some estimates for the costs of water transport provided by different studies can be found in Herzog (2014a).

Animals play different roles in path creation: according to Lay (1992, pp. 6–7), the first human ways had an animal path origin (cf. Whitley & Burns, 2007). Moreover in some areas special paths for herds existed in the past. Horse riding, pack or draft animals also had an impact on the velocity of the traveller. It is very difficult to find appropriate cost functions taking animal movement into account due to the large variety within each species (e.g. oxen or horses) and the high number of possible species to be considered (Ganskopp & Vavra, 1987).

Creating the accumulated cost surface (ACS)

In the early days of LCSC and LCP applications in archaeology, these studies were mainly based on an isotropic cost grid. Such a cost grid stores for each cell the costs of traversing the cell independent of the direction of movement. In this case, the costs of movement from a cell to one of its four direct neighbours is the average of the costs of the start and the target cell. If the movement is diagonal to a corner-connected cell, the length of the move must be taken into account, i.e. the average of the costs has to be multiplied by √2 (Figure 18.3).

3 x 3 cost grid

8	8	80
10	**10**	90
12	16	100

ACS

√2*0.5*(10+8)	0.5*(10+8)	√2*0.5*(10+80)
0.5*(10+10)	0	0.5*(10+90)
√2*0.5*(10+12)	0.5*(10+16)	√2*0.5*(10+100)

Figure 18.3: Simple example of an isotropic cost grid (left) and the corresponding ACS (right). The origin of the accumulation process is the centre of the cost grid (cost value = 10, bold).

The cell centres and the possible moves to the neighbouring cell centres form a graph. For graphs, efficient algorithms calculating the least-cost route from a given origin to all other nodes (i.e. cell centres) are known if all cost distances are positive (Figure 18.4 is based on Cormen, Leiserson, Rivest, & Stein, 2001, pp. 476–495, 595–599; Dijkstra, 1959).

For LCSCs, the algorithm has to be modified: in Step 3 only those cell centres are inserted in the candidate set, whose ACS value is below the predefined cost limit.

For LCP generation modifications of the algorithm are also needed. Firstly, whenever a new ACS value is assigned to a cell centre in Step 3, the backlink for this cell is stored, i.e. the current position. Secondly, if the target of the LCP is selected in Step 2, the LCP is generated by connecting the backlinks from the target to the origin. An optional procedure may save computation time: Initially the costs of the most direct connection to the target may be calculated. Only those possible candidates should be inserted in the set, for which the sum of the current ACS value and the minimum costs for the straight-line distance to the target are below this initial cost limit.

The results of the LCP algorithm is independent of the units of measurement chosen, i.e. they do not change if all costs are multiplied by a constant factor. So applying the multiplier of 0.8 for horse riding suggested by Tobler (1993) will result in the same calculated paths as the initial formula for hikers. Similarly, the LCPs generated for male and female walkers based on one of the formulas proposed by Irmischer and Clarke (2017; no. 3 in Table 18.2) do not differ.

Both the conversion of vector data to a cost raster and the subsequent conversion of this raster to a graph may produce unexpected results. These issues are illustrated in Figures 18.5 and 18.6. Figure 18.5a shows an isotropic cost grid with a linear barrier (cost value of 100) in an area of uniform costs (white cells are assigned a cost value of 10). The grey cells indicate three ways of converting the barrier to raster cell values: including all cells whose centre is within a 5, 7.5 and 10 m distance from the line.

Three ways of converting grid cells to a graph have been used in archaeological least-cost calculations: (i) linking each cell with its 8 nearest neighbours (queen moves in Figure 18.5b-d), (ii) ensuring that all cells within a 24 cell neighbourhood can be reached without detour (queen and knight moves), or (iii) connecting to cells in all directions in a 48 cell neighbourhood (all lines starting from the origin in Figure 18.5b).

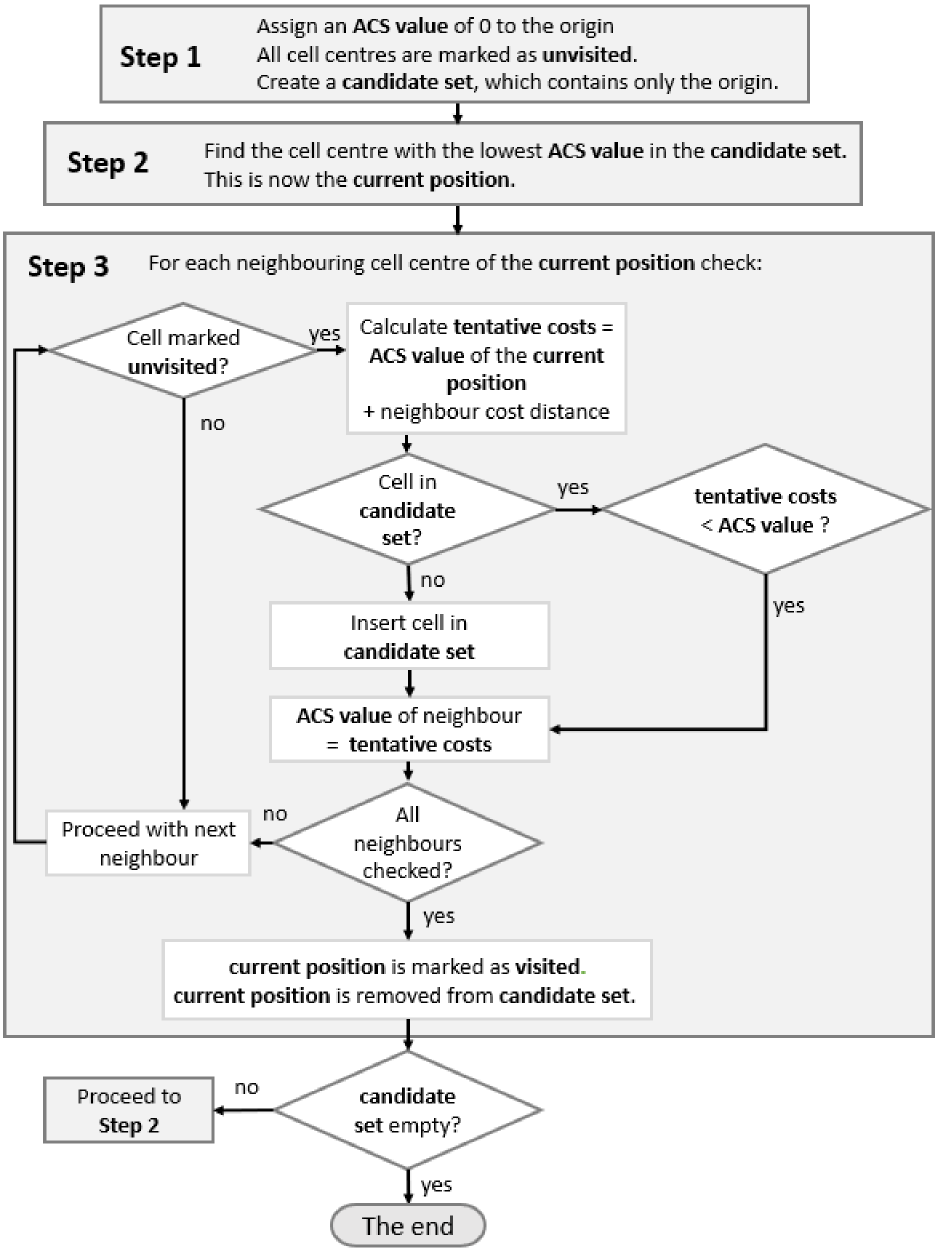


Figure 18.4: Dijkstra’s algorithm applied for a cost grid

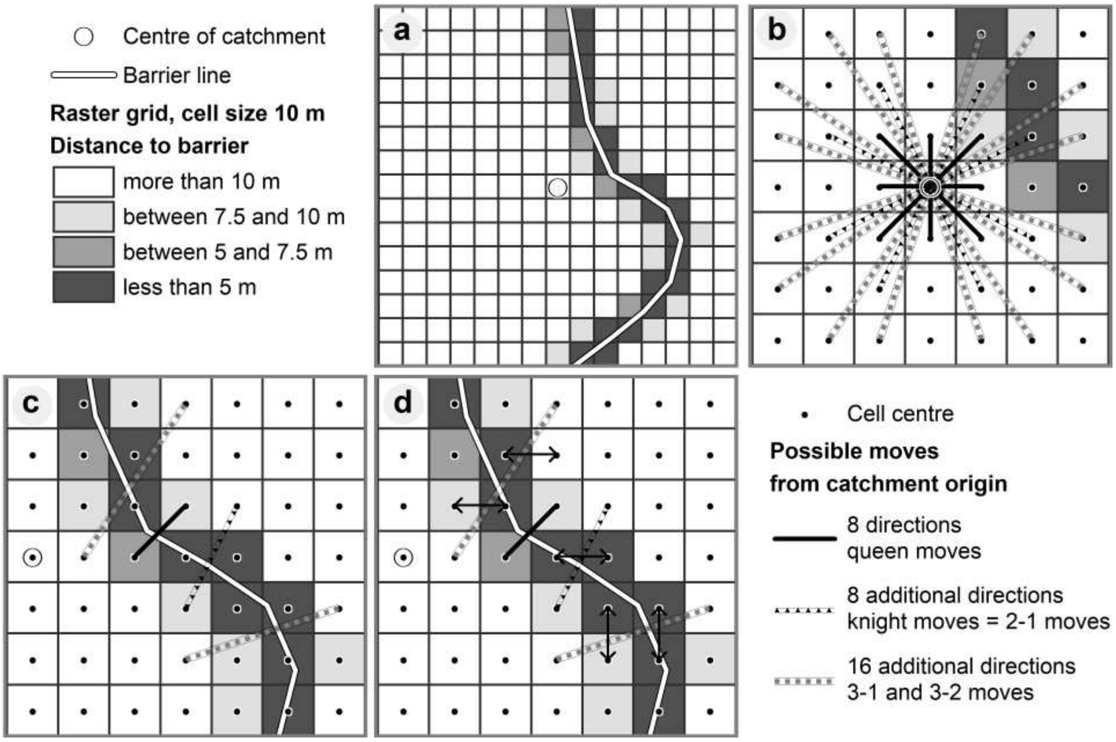


Figure 18.5: (a) simple isotropic cost grid, (b) possible moves starting at the origin, (c) traversing the barrier cells by long moves, (d) subdividing long moves.

Figure 18.5c illustrates how a simple diagonal move may traverse the 5 m radius barrier without paying due costs; knight moves can jump over the 7.5 m radius barrier, and the long 3–1 and 3–2 moves cross the 10 m radius barrier without touchdown on a high cost cell. This issue can be avoided by cutting the long moves into two or three sub-moves respectively, as indicated by the lines with arrows in Figure 18.5d. The cost values of the cut points are the weighted averages of the two values stored in the cells connected by the arrow lines, with the weights depending on the distance to the cut point.

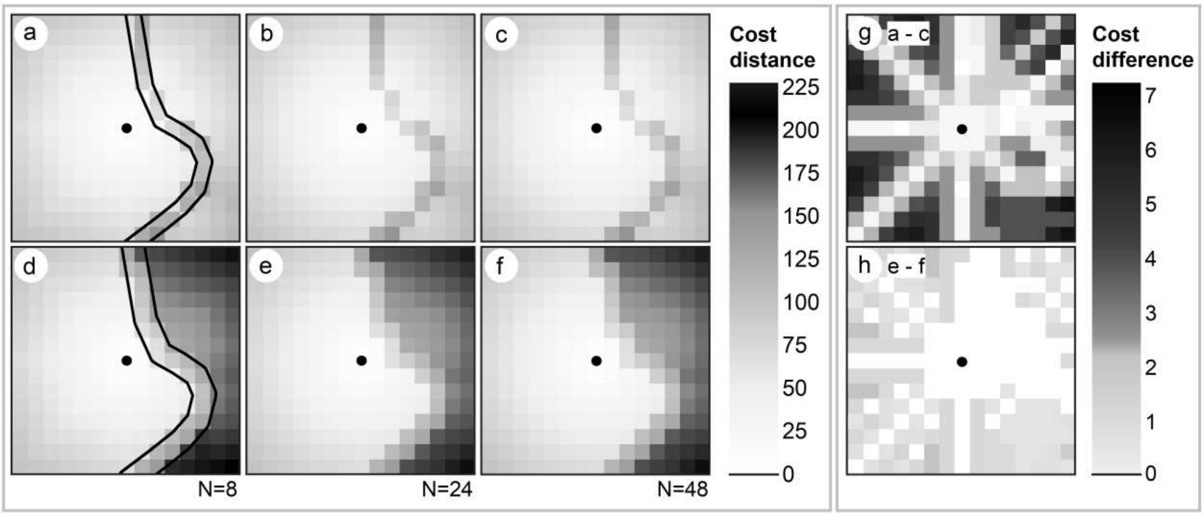
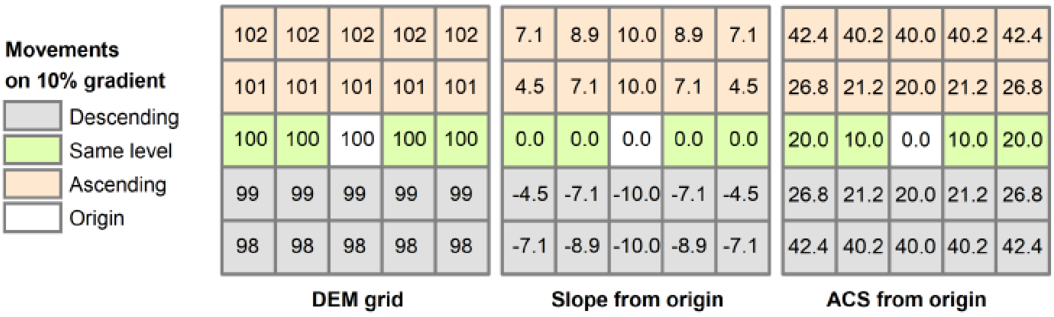


Figure 18.6: a-f depict ACS results based on the cost grid shown in Figure 18.5a. The outcomes of an inadequate barrier radius of merely 5 m are shown in a-c. For d-f an adequate barrier radius of 7.5 m was chosen. The N values indicate the number of nearest neighbouring cells that can be reached from the origin without detour. Images g and h illustrate the impact of different N values by grids showing the differences in accumulated costs.

Figure 18.6a-c depicts the ACS based on the cost grid with the 5 m radius barrier shown in Figure 18.5a. Due to the diagonal moves that jump over the barrier, some ACS cells beyond the barrier have a lower accumulated cost value than any barrier cell. This unwanted effect is avoided with the 7.5 m barrier (Figure 18.6d-f). The cut point implementation of the long moves ensures that due costs are paid for the barrier. The correct shortest paths from the origin to the cells in the west half of the cost grid are straight lines, so that cost distance and straight line distance should coincide in this area, i.e. cells of equal cost distance from the origin should form a semicircle in the west. The image for N=48 shows more of a semi-circular structure than the N=8 image. In fact, by increasing the number of move directions, the detours necessary for reaching the target locations in general are made smaller. This is illustrated by the difference images in Figure 18.6g and h: in the uniform terrain area the largest difference is about 6 m, the distance of the corresponding cells to the origin is about 77 m. So with N=8, the largest detour is about 7.8% of the true shortest distance. This elongation error decreases substantially for N=24 (Figure 18.6h): in the uniform area, it is about 1.5 m on covering a straight line distance of 63 m, i.e. about 2.4% of the true shortest distance (Herzog, 2013b).

Various algorithms for calculating slope exist (Conolly & Lake, 2006, pp. 191–192; Lock & Pouncett, 2010; Wheatley & Gillings, 2002, 120–121) providing different outcomes. Moreover, confusion of units for measuring slope may result in unrealistic ACS grids. This is why deriving slope directly from the DEM in the process of ACS calculation is recommended as illustrated in Figure 18.7.



DEM grid

102	102	102	102	102
101	101	101	101	101
100	100	100	100	100
99	99	99	99	99
98	98	98	98	98

Slope from origin

7.1	8.9	10.0	8.9	7.1
4.5	7.1	10.0	7.1	4.5
0.0	0.0	0.0	0.0	0.0
-4.5	-7.1	-10.0	-7.1	-4.5
-7.1	-8.9	-10.0	-8.9	-7.1

ACS from origin

42.4	40.2	40.0	40.2	42.4
26.8	21.2	20.0	21.2	26.8
20.0	10.0	0.0	10.0	20.0
26.8	21.2	20.0	21.2	26.8
42.4	40.2	40.0	40.2	42.4

Figure 18.7: Small DEM with a cell size of 10 m and a constant slope value of 10% with the corresponding ACS for the cost function $Q(\hat{s}) = 1 + (\hat{s} / \check{s})^2$, with $\check{s} = 10$ (cf. no. 9 in Table 18.2).

Even with an isotropic slope-dependent cost function, the direction of movement is important. The move from the centre cell with an altitude of 100 to the north has a slope of 10% accumulating costs of 2, whereas the moves to the east or west remain at the same altitude accumulating only half of the costs. The moves to the diagonal cells are longer, resulting in a lower slope than for the cells to the main directions ($\hat{s} = 10/\sqrt{2}$), but still a higher accumulated cost value. Figure 18.8 presents another visualisation of anisotropic movement on different gradients with constant slope and different cost functions.

According to the approach presented in Figure 18.7, movement on the contour line of a DEM is equivalent to the movement on level ground. For contour lines of a steep gradient, this is only true if some construction work has been done to create the path (Figure 18.9). For informal routes that did not involve any building effort, very steep gradients may be considered as barriers. It would be important to note that with the exception of contour line routes, construction work such as removal of outcrops, building bridges and tunnels is mostly not included in ACS generation. Clearly, the outcome of the approach outlined above depends on the accuracy and resolution of the DEM (Herzog & Posluschny, 2011).

An anisotropic cost grid may be combined with an isotropic grid by multiplying or adding accumulated cost values (Herzog, 2013a). The terrain factors listed in Table 18.3 suggest multiplication, and multiplication is independent of the resolution of the cost grids. However, several authors prefer adding cost components (e.g. Fovet & Zakšek, 2014), often a weighted sum of cost grids is created (e.g. Whitley & Burns, 2008). For modelling anisotropic costs such as currents or wind directions, more complex approaches are required (Collischonn & Pilar, 2000; Indruszewski & Barton, 2005, 2007).

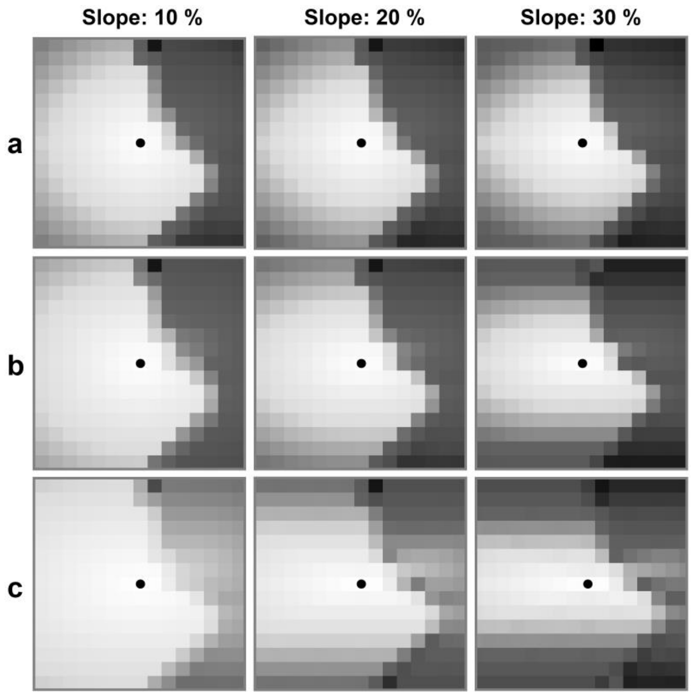


Figure 18.8: ACS for different slope dependent cost functions (nos. 1, 6, and 9 in Table 18.2) on three gradients, with an additional barrier as in Figure 18.5a (radius 7.5 m). For each cost function, the costs vary from 0 at the origin, depicted in white, to the largest accumulated cost value depicted in black. a) Ericson & Goldstein b) Tobler c) Q(12).

Figure 18.9: Built level path on a steep slope in the hilly area east of Cologne, Germany.

After deciding on the relevant cost model, the main difficulty for LCSC generation is the decision on a cost limit. For single farmsteads with crop-based economy the farm sizes listed in the study by Kerig (2008) may provide some guidance: the minimum farmland is 2 hectare, and the maximum is 4 to 5 hectare if all work is to be carried out by humans. With oxen larger farmlands of up to 10 hectare can be ploughed. Horses allow ploughing even larger plots, up to 33 hectare.

Figure 18.10 clearly shows that the LCP may deviate from the true shortest path depending on N (for details see Herzog, 2013b). By increasing N, calculations will be rendered more accurate but the computation time will increase as well. The figure shows LCPs created with Dijkstra's algorithm for paths in both directions, based on the ACS presented in Figure 18.6d-f. In an area of uniform costs such as the western half of the cost grid used in Figure 18.6, the optimal path is a straight line. But for N=8 and uniform costs, only the LCPs in the eight directions considered coincide with the straight line, an example is the LCP to target no. 3. But for paths in other directions such as to target no. 5, the LCP deviates from the correct shortest path (dotted line in Figure 18.10a), this deviation decreases when increasing N. Moreover, two different LCPs are depicted for most targets: the return path accumulates the same costs.

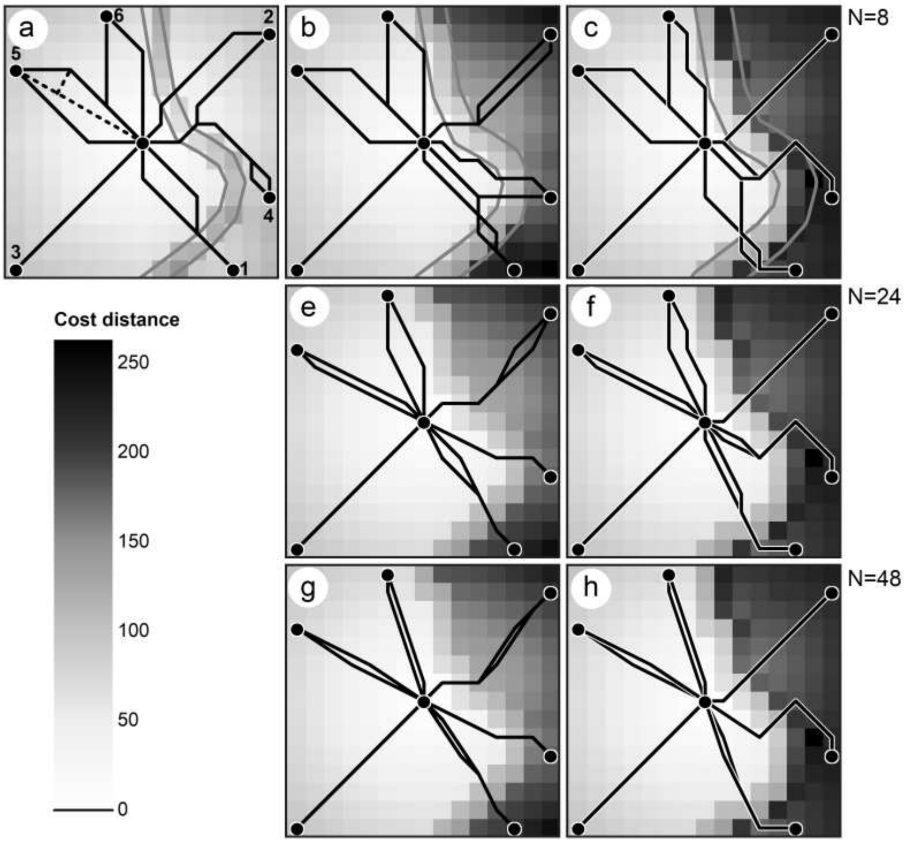


Figure 18.10: LCPs (black lines) from the origin in the centre to five different targets. The outcome of the LCP algorithm depends on the number of nearest neighbours that can be reached without detour (N) and the width of the barrier (a: width = 5 m; b, e, g: width = 7.5 m; c, f, h: width = 10 m; cf. Figure 18.6).

Case-study

In the hilly rural area southwest of Cologne quite a few Roman sites including the remains of farms (*villae rusticae*), temples, and roads have been recorded. The aim of the case study is to compare the site catchments of a farm (villa in Blankenheim) and a temple (known as Görresburg). The cost model derived from the Roman roads in this area is applied for the catchments. Although the movement patterns of travelling on a Roman road may differ from small-scale movement within a site catchment, no data concerning the latter is readily available. The first step is to identify the principal factors governing the construction of Roman roads in this area (Figure 18.11).

In this study area, large parts of the Roman road known as the Agrippa Road have been recorded by aerial photography, ALS data and some small-scale excavations (Grewe, 2004, 2007; Horn, 2014, map p. 169; www.erlebnisraum-roemerstrasse.de/stationen/). Another Roman road section in this area was proposed and verified by Hagen (1931, p. 176). The Roman road section suggested by Schneider (1879, p. 21) relies mainly on straight-line sections of roads that were still in use in the mid-19th century and passes a known Roman road site, therefore it is also tentatively included in this set of Roman roads. Unfortunately, landscape reconstruction is beyond the scope of this small case study, so Figure 18.11 shows some modern features, mostly roads such as a modern motorway east of the site labelled “Roman road remains” in the north-east of the map.

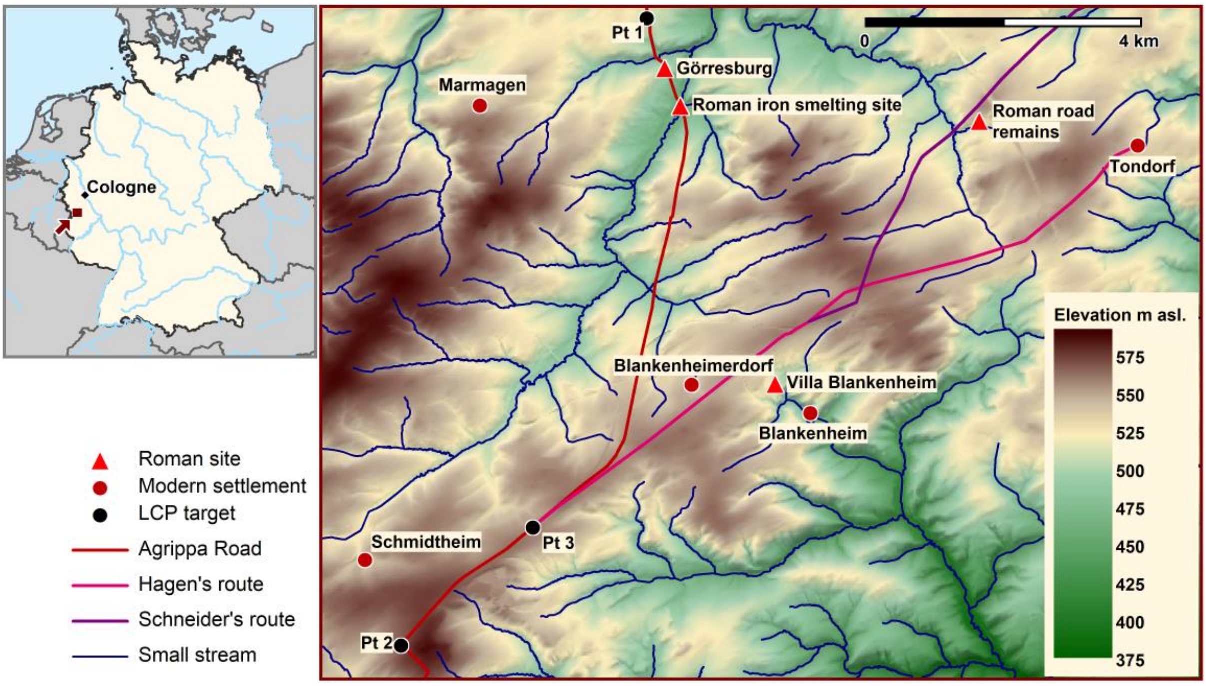


Figure 18.11: The study area south-west of Cologne covering approximately 13 by 10 km

When discussing the Agrippa Road, Grewe (2004) pointed out that Roman roads avoided steep slopes in order to allow horse or oxen driven carts to proceed. According to Grewe, the slopes of Roman roads in the Rhineland normally do not exceed 8 percent although at some exceptional locations 16 to 20 percent have been recorded. Due to the slope restrictions for Roman roads, the cost factor slope is an obvious choice. Slope is derived from the two DEMs available for the study area (Table 18.4).

Name	Cell size	Projection (EPSG)	Data collection	Median slope
DEM10	10 m	Gauss-Krüger (31466)	Photogrammetry, ALS	8.4%
DEM25	25 m	ETRS89 (25832)	ALS	8.0%

Table 18.4. DEM data provided by the ordnance survey institution (Geobasis NRW) responsible for this part of Germany

Experience with another hilly region in the Rhineland suggested including another cost factor that models streams as barriers (Herzog, 2013a, 2013b, 2013c, 2013e). This was tested for DEM10: a buffer with a radius of 7.5 m was created for the streams and isotropic costs of 5 assigned to the cell centres within the buffer (Figure 18.12: iso = 5). All streams in the neighbourhood of the Roman routes considered belong to the class “width below 3 m”, therefore a uniform penalty for traversing streams is considered appropriate. The LCPs derived from this cost model agree quite well with the Roman road proposed by Hagen, but deviate from the route suggested by Schneider. With respect to the Agrippa Road, the results are not very convincing. Modifying the penalty for crossing streams does not change the outcome, because the number of stream crossings for the initial LCPs and the Agrippa Road is about the same. An alternative is a model derived from the soil map that takes the wet soils in the stream valleys into account. Moreover, ford or bridge locations were digitized from the map set created in the years 1846–1847 and assigned costs of 2. Based on this model, the LCP generated from the slope-dependent cost function with a critical slope of 6 % reconstructs the Agrippa Road somewhat better than the other LCPs (Figure 18.12).

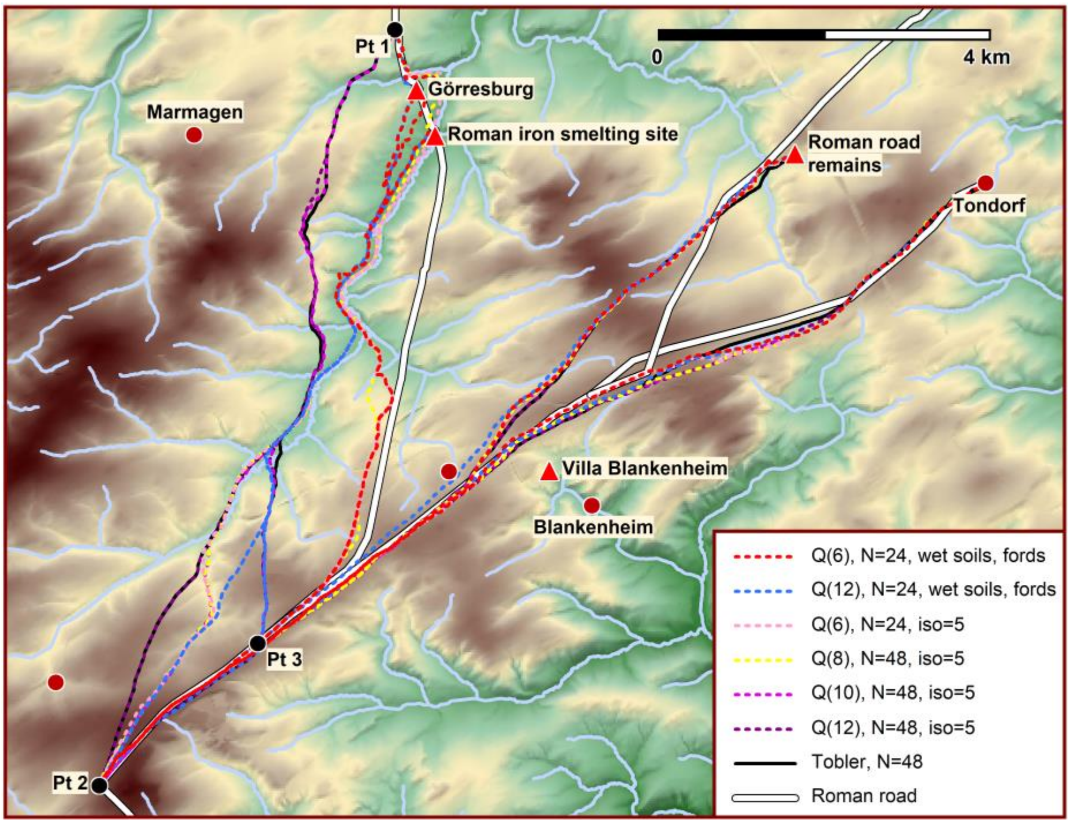


Figure 18.12: LCPs based on the formulas by Tobler, Irmischer, and Herzog/Minetti (see nos. 1, 3, and 11 in Table 18.2) as well as quadratic slope dependent cost functions (see no. 9 in Table 18.2) combined with costs for traversing water courses and/or wet soils.

Moreover, the LCPs based on Tobler's hiking function were generated, and though they do not pay penalties for crossing water, they agree quite well with some of those derived from the vehicle cost function with streams modelled as barriers. The LCPs generated from a cost model combining the Irmischer off-road cost function with penalties for wet soils except at ford locations are often more direct than the rest of the LCPs presented and often do not reconstruct the known roads as well as these.

After the first sobering results, Görresburg and the Roman smelting site were included as additional possible origins besides Pt1 (cf. Figure 18.11). This allows testing if the road made a detour on purpose to pass these sites.

			Prominence: DEM10, 100 m radius				Prominence: DEM25, 250 m radius			
	Length (km)	height change	-6.3 to -1.0	-1.0 to 0.0	0.0 to 1.0	1.0 to 5.2	-15.0 to -2.0	-2.0 to 0.0	0.0 to 2.0	2.0 to 13.4
Agrippa	10.87	432 m	11	24	36	30	12	8	29	46
Q(10)	10.67	266 m	43	21	33	4	52	19	18	12

Table 18.5: Comparison of the Agrippa Road section and the LCP generated based on the cost function Q(10) with a critical slope of 10% (see no. 9 in Table 18.2) combined with a penalty factor of 5 for crossing streams. For the two routes, the percentage in each prominence category is given.

The Agrippa Road and the Q(10) LCP are of similar length, but the total of elevation differences derived from a trail elevation profile is considerably lower for the LCP (height change in Table 18.5). So neither minimising height change nor avoiding crossing streams or wet soils are the principal factors governing the construction of the Agrippa Road. Long sections of the Q(10) LCP run in the stream valleys, whereas the Agrippa Road after crossing a stream immediately climbs to more elevated terrain (Figure 18.13a). Viewsheds probably did not play an important role in this forest area, but a method for calculating local visual prominence can be applied to altitude data for identifying elevated areas (Llobera, 2003; Figure 18.13b).

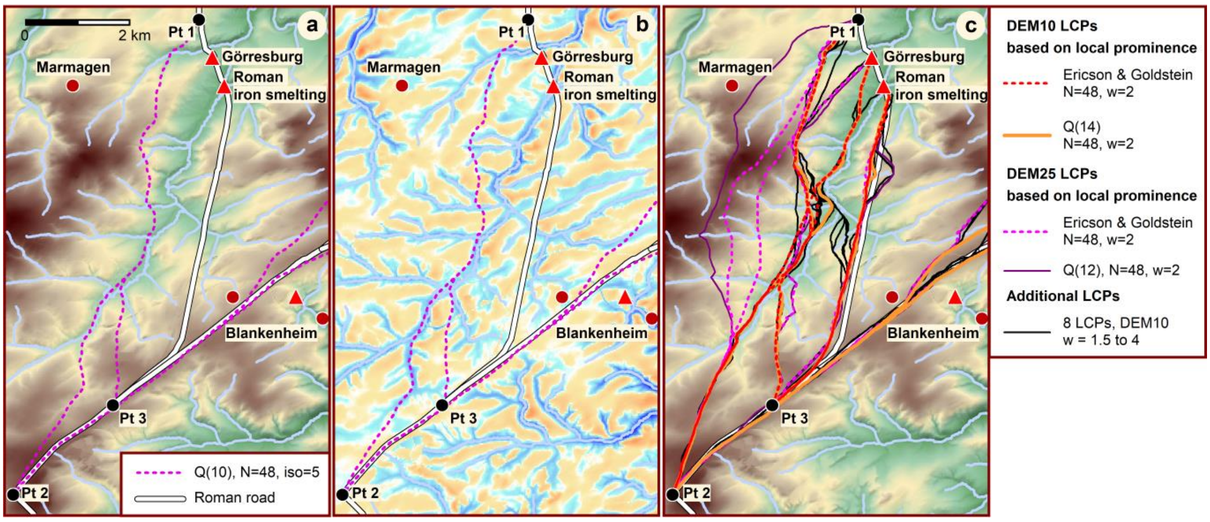


Figure 18.13: a) Comparison of the Q(10) LCPs with the Agrippa road, b) comparison of the local prominence for these two routes (white = low, black = high prominence) c) LCPs with increased isotropic costs in areas of low prominence.

Table 18.5 clearly shows that the Agrippa Road avoids areas of low prominence. Therefore LCPs with different cost multipliers (w values in Figure 18.13c) attributed to low prominence areas were calculated. The cost multiplier 2 generated the best results combined with slope-dependent cost functions that assign less costs to steep slopes than Q(10). Such cost functions tend to generate straight road sections typical for Roman roads. But omitting the slope-dependent cost component produces LCPs that are not as close to the Agrippa Road as the LCPs highlighted in Figure 18.13c.

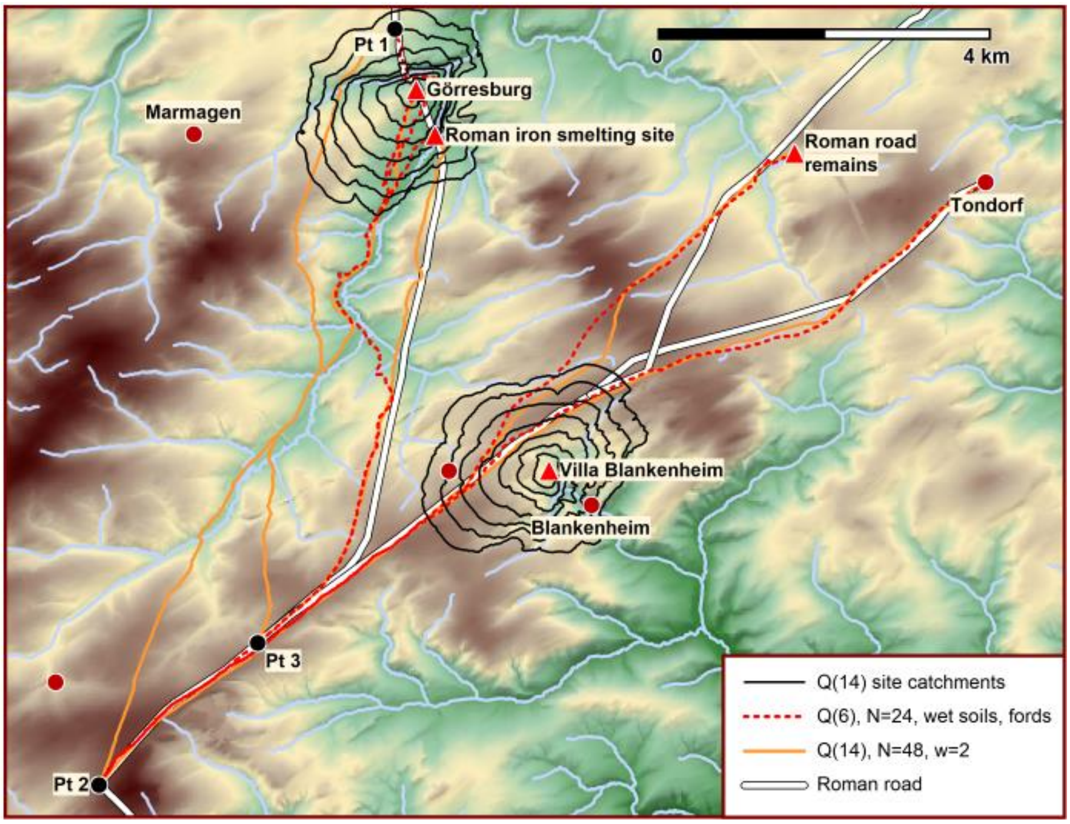


Figure 18.14: The best performing LCPs for the models considered and the LCSCs derived from the Q(14) cost model for the Görresburg temple and the villa.

Only the LCPs starting at the Roman iron smelting site coincide well with the Agrippa Road suggesting that this site determined the layout of the road to some extent. Figure 18.13c shows also that the choice of the DEM can have an impact on the result when considering the LCPs connecting Pt1 and Pt2. However, the more important LCPs connecting the Roman iron smelting site with Pt3 coincide quite well independent of the DEM chosen.

Based on the Q(14) cost function and a cost multiplier of 2 for low prominence areas, LCSCs were calculated for the temple on the Görresburg and the villa near Blankenheim (Figure 18.14). Cost limits are measured in terms of walking on level ground, without stepping into low prominence areas and are given in multiples of 250 m. With respect to sizes, the two different sets of catchments do not differ substantially (Table 18.6). South of the Görresburg hill, excavations found a Roman settlement (Horn, 2014, pp. 196–197), so agricultural use was probably important for both locations.

LC-distance	250m	500m	750m	1000m	1250m	1500m	1750m
Temple	10.3	33.2	69.3	117.2	180.2	272.4	405.2
Villa	9.9	29.4	63.6	121.5	207.5	316.3	448.7

Table 18.6: Areas included within the LCSCs in hectares.

This case study covering only a small area mainly suggests hypotheses to be tested in larger areas with a larger number of Roman sites. It should be noted that the cost model for Roman roads found in this example bears similarity with that found by Verhagen and Jeneson (2012) dealing with a Dutch Roman road section close to the German border.

Conclusion

Validation, assessing the accuracy, and analysis of the stability of the outcomes

For a convincing application of a cost function for LCSC or LCP generation, analysing the archaeological or historical evidence and some validation is required. For instance, Garmy et al. (2005) mention that the cost function chosen reproduces already known old footpaths in their study region.

GPS trails (Márquez-Pérez et al., 2017) and walking experiments (Kondo et al., 2011) can provide some data for validating the cost function applied in the study area considered, but in general, modern people are not as used to walking as people in past times. Energy expenditure might be overestimated by cost functions based on modern measurements because the energy consumption of walking or running is lower for people used to walking or running most of the day compared to that of modern white collar workers (Pontzer et al., 2012).

If the aim of an LCP study is to reconstruct a known road, the similarity between the LCP to the known route can be assessed by determining the proportion of the LCP that lies within a buffer distance from the known road (Goodchild & Hunter, 1997). Applications of this simple measure of similarity in archaeological LCP studies were published by Canosa-Betés (2016), Güimil-Fariña and Parcero-Oubiña (2015) as well as Lynch and Parcero-Oubiña (2017).

If the location of the roads to be reconstructed is not known, validation often relies on road indicator sites, for example grave monuments or mile stones that can be found close to Roman roads (e.g. Güimil-Fariña & Parcero-Oubiña, 2015). On a larger scale, archaeologists often assume that settlements are located close to main roads. For instance, Lynch and Parcero-Oubiña (2017) calculated the distance from each site in their study area to the closest calculated path. If the road indicator sites are clustered (e.g. Güimil-Fariña & Parcero-Oubiña, 2015) statistical tests relying on independent observations are problematic. Therefore validation based on such sites is not straight-forward.

Finally, validation of LCP results by survey is another possibility (e.g. Rademaker et al., 2012; Rogers et al., 2015). However, the number of finds in the vicinity of old routes that can be discovered by field walking is limited, Rogers et al. (2015) detected only one artefact that was older than 200 years during two days of prospection. The remains of minor roads such as sunken lanes may lead to inadequate conclusions. Moreover, continuous use of routes until today and stray finds are issues in LCP validation by field survey.

Some conclusions

A wide variety of cost functions for walkers is available, but validated cost functions for the movement of pack or draft animals as well as for water transport can rarely be found. Moreover, footpaths following animal tracks might exhibit a large variation of preferred slopes, because the study by Ganskopp and Vavra (1987) shows that different species prefer different slopes: the average slopes of sites utilised by cattle, feral horses, mule deer, and bighorn were 5.8, 11.2, 15.7, and 42.5% within one study area. For non-pedestrian transport further research is required to provide reliable cost functions.

Some authors of archaeological LCP studies believe that the selection of the cost model is of minor importance (Bellavia, 2002; Verhagen & Jeneson, 2012). But many publications present quite different LCP results for different slope-dependent cost functions (e.g. Canosa-Betés, 2016; Güimil-Fariña & Parcero-Oubiña, 2015; Rademaker et al., 2012, Plate 2). Often, LCPs derived from several cost models coincide only in areas where this route is the obvious choice such as mountain passes or flat areas.

Most archaeological LCP and LCSC studies rely on software created by somebody else. Gietl, Doneus, and Fera (2008) showed some time ago that it is often not possible to recreate the LCP results of one software package with another. Some of the LCP software used by Gietl and his colleagues has been improved in the last decade, but there are still substantial differences in their potential for modelling anisotropic friction and movement steps in more than eight directions.

Cost function based approaches beyond LCPs and LCSCs

This contribution discussed LCPs connecting point pairs. Different concepts of connecting dots by a network of routes exist, depending on the frequency a route is used and the effort required to construct roads or paths. Overviews of approaches for connecting a set of points are presented in Herzog (2013c) as well as Groenhuijzen and Verhagen (2017).

Several least-cost approaches have been proposed for identifying corridors of movement: adding the two ACS for two locations results in a raster with low values where progress is easy. The low value cells indicate possible corridors of movement between the two locations (e.g. Palmisano, 2017).

Adding LCSC for each cell in the study area, independent of selected target locations is an approach for calculating the accessibility of each cell (Mlekuz, 2013). This and additional methods for calculating accessibility based on cost functions are discussed in Herzog (2013d). An approach for identifying zones of high accessibility based on focal mobility networks for random points and kernel density estimation of cells visited frequently by the focal paths was presented by Canosa-Betés (2016). Verhagen (2013) suggested integrating indicators of accessibility based on least-cost models in a predictive modelling framework. A method for avoiding overlapping site catchments is to stop the spreading process whenever two site catchments meet, this is resulting in the generation of least-cost Thiessen polygons (Herzog, 2013c).

People who walk into unknown territory see only part of the landscape ahead whereas LCPs are based on the total knowledge of the landscape ahead. For modelling dispersal processes into unknown terrain starting from a given location, paths consisting of locally optimal steps in the direction chosen initially may be generated. An agent-based algorithm for modelling such dispersal processes was proposed by Herzog (2016). The approach presented by Lock and Pouncett (2010) is also based on progress in local neighbourhoods and introduces the term “corridor of intentionality” in this context. Moreover, they point out the importance of cultural landscape features as mid-distance waypoints.

LCP technology can also be applied to reconstruct Roman long-distance water supply systems (Orengo & Miró i Alaix, 2013). Wood and Wood (2006) suggest applying least-cost distances for economic modelling in archaeology, assuming that "artifacts derived from a particular resource will inversely correlate with the energetic distance from the origin of that resource". Least-cost approaches for kernel density estimation (Herzog & Yépez, 2013) and Ripley's K (Negre, Muñoz, & Barcelo, 2017) have been applied in archaeological studies. In fact, any method of spatial statistics relying on Euclidian distances can be modified so that another distance measure is used. But it is important to remember that least-cost distances in general are no mathematical distances, because the path from A to B may involve different costs than the return path from B to A, so that cost functions based on averaging the costs in both directions should be used when modifying an algorithm designed for straight-line distances by using cost distances.

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